Periodic steady-state heat transfer in cooling panels

K. A. Antonopoulos and F. Democritou
Mechanical Engineering Department, Thermal Section, National Technical University of Athens, Athens, Greece

A slab is considered to be exchanging heat internally with a fluid flowing through pipes embedded in the slab and externally by convection and radiation with the ambient surroundings in a periodic fashion. In practice, such cases are encountered in panel cooling of buildings, in which the heat produced within or inserted into a room is absorbed by a cold fluid flowing through pipes imbedded in the roof slab of the room. A method is developed for predicting the performance of such systems, which is based on a finite-difference solution of the transient two-dimensional heat conduction differential equation within the roof slab containing the cooling pipes. The boundary conditions on the upper surface are periodic, i.e., it is assumed that the daily cycle of the ambient temperature and the incident solar radiation are repeated on consecutive days, thus leading to a periodic steady-state solution. In the case of floor slabs, the problem becomes steady-state unless intermittent operation is to be examined. A parametric study is presented, in which the effect is examined of the most important parameters, including the pipe spacing and the cooling fluid temperature.

Keywords: panel cooling; periodic heat transfer

Introduction

In this paper, panel cooling of buildings is considered, in which the heat produced within or inserted into a room is absorbed by a cold fluid flowing through pipes imbedded in the roof slab of the room. This method offers known advantages over the conventional space-cooling systems.

In previous studies the problem has been examined in an oversimplified way by using mainly steady-state one-dimensional (1-D) solutions. In the present work, a procedure and a corresponding computer code is developed for the calculation and analysis of space panel cooling systems, which are based on a finite-difference solution of the transient two-dimensional (2-D) heat-conduction differential equation within the roof slab containing the cooling pipes. In the general case, the boundary conditions on the upper surface of the roof slab are periodic, i.e., it is assumed that the daily cycle of the ambient temperature and the incident solar radiation are repeated on consecutive days. Therefore, the solution converges to a periodic steady-state condition, i.e., it is repeated on consecutive days. Such solutions have been obtained for characteristic days of the year and for typical roof-slab constructions, using the climatological conditions for the Athens area.

In the case of floor slabs (i.e., those separating two stories), the problem is simplified because the air temperature at the upper surface of the slab is constant and solar radiation is absent. Therefore, the problem can be treated as steady-state, unless the case of intermittent operation is to be examined.

Address reprint requests to Professor Antonopoulos at the Mechanical Engineering Department, Thermal Section, National Technical University of Athens, 42 Patission St., Athens 106 82, Greece.

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Steady-state solutions have been obtained for typical floor-slab constructions.

A parametric study has also been conducted, in which the effect has been examined of the important parameters, including the mean cooling fluid temperature, T_f, and the pipe spacing, S, i.e., the distance between the axes of the cooling pipes.

Differential equation and boundary conditions

With reference to Figure 1, the 2-D transient heat-conduction equation may be written as

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) \tag{1}$$

where $T(t, x, y)$ is the temperature; t, x, y denote the time and space coordinates; and ρ, c, k stand for the density, the specific heat, and the thermal conductivity, respectively, of the various layers of the roof slab. Differential equation (1) may be solved within the unit of symmetry of the panel system OABC (i.e., for $0 \leqslant x \leqslant S$, $0 \leqslant y \leqslant L$) or ODEC (i.e., for $0 \leqslant x \leqslant S/2$, $0 \leqslant y \leqslant L$). The boundary conditions are described below.

On the lower surface OA, the heat flow $\dot{q}_l$ is prescribed, i.e.,

$$\dot{q}_l(t, x, 0) = h_l[T_a(t) - T_l(t, x, 0)] + h_{r,l}[T_w(t) - T_l(t, x, 0)] \tag{2}$$

where

$T_a(t)$ = room air temperature, °C

$T_w(t)$ = mean temperature of wall and floor surfaces, °C

$T_l(t, x, 0)$ = lower surface temperature, °C

h_l = coefficient of heat transfer by convection at the lower surface of the roof slab, W/m^2 °C

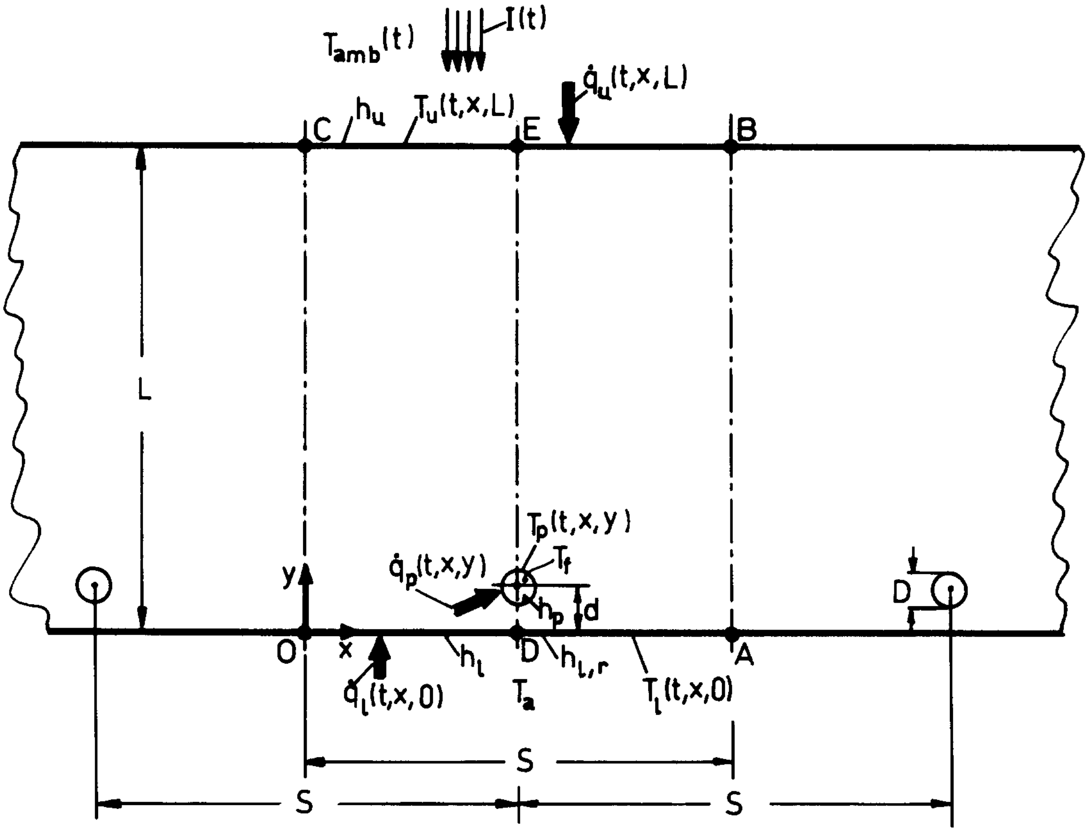

Figure 1 Cross section of panel system and solution domain OABC

$h_{r,l}$ = coefficient of heat transfer by radiation at the lower surface of the roof slab

On the upper surface CB, the heat flow $\dot{q}_u$ is prescribed, i.e.,

$$\dot{q}_u(t, x, L) = aI(t) + h_u[T_{amb}(t) - T_u(t, x, L)] - \varepsilon\Delta R \qquad (3)$$

where

a = absorptance of the upper surface for solar radiation

$I(t)$ = total solar radiation incident on the upper surface, W/m^2

h_u = coefficient of heat transfer by long-wave radiation and convection at the upper surface, $W/m^2\ ^\circ C$

$T_{amb}(t)$ = ambient air temperature, °C

$T_u(t, x, L)$ = upper surface temperature, °C

ε = hemispherical emittance of the upper surface

ΔR = difference between the long-wave radiation incident on the surface from the sky and surroundings and the radiation emitted by a black-body at ambient air temperature, W/m^2

Notation

a	Absorptance for solar radiation
a_i, b_i, c_i	Coefficients in Equations 8 to 10
A, B, C	Solar parameters in Equation 7
c	Specific heat
C_i	Coefficient in Equation 11
d	Depth of pipe within slab
D	Pipe diameter
h	Heat transfer coefficient
I	Total solar radiation
J	Day of the year
k	Thermal conductivity
L	Roof or floor slab thickness
M	Coefficient in Equation 11
$\dot{q}$	Heat flow
S	Pipe spacing
S_i	Coefficient in Equation 11
t	Time
T	Temperature
x, y	Space coordinates

Greek symbols

β	Solar altitude
$\varepsilon\Delta R$	Correction term in Equation 3
ρ	Density

Subscripts

a	Room air
amb	Ambient air
f	Fluid
l	Lower surface
p	Pipe
r	Radiation
u	Upper surface
w	Wall

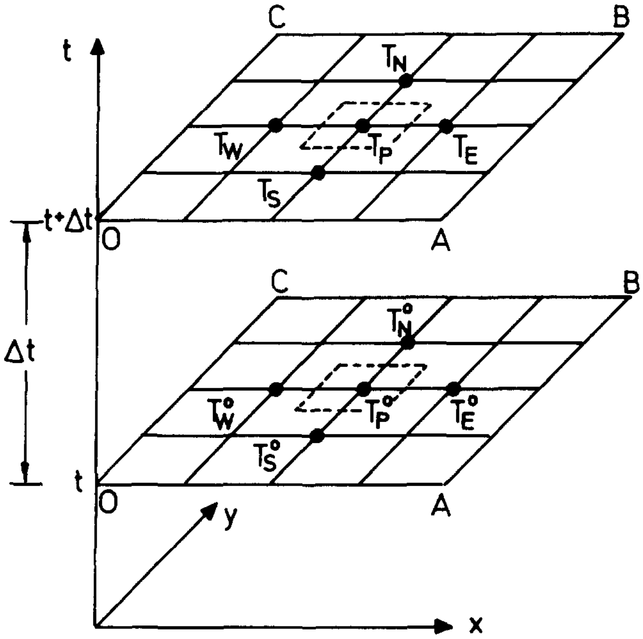

Figure 2 Simplified grid of coordinate lines *x-y-t* for the finite-difference solution of differential equation 1

On the interior pipe surface, the heat flow $\dot{q}_p$ is prescribed, i.e.,

$$\dot{q}_p(t, x, y) = h_p[T_p(t, x, y) - T_f] \quad (4)$$

where

h_p = coefficient of heat transfer by convection at the interior surface of the pipe, W/m^2 °C

$T_p(t, x, y)$ = interior pipe surface temperature, °C

T_f = mean cooling fluid temperature, °C

The assumption of constant fluid temperature, equal to the mean fluid temperature T_f, is close to reality because the change of fluid temperature as the fluid passes through the roof slab from one wall of the room to the other is very small. For example, it can be easily calculated that for a panel cooling power of 100 W/m^2 with pipe spacing and diameter $S = 0.20$ m and $D = 0.02$ m, respectively, and a fluid velocity of 0.1 m/s, the resulting fluid temperature change is 0.15°C/m.

On the boundaries OC and AB, which are planes of symmetry, the following boundary conditions are imposed, respectively:

$$\frac{\partial T}{\partial x}(t, 0, y) = 0 \qquad \frac{\partial T}{\partial x}(t, S, y) = 0 \quad (5)$$

Solution procedure for the periodic steady-state condition

Solution of the differential equation (Equation 1) is obtained within the domain OABC of Figure 1 by employing the finite-difference procedure described in Patankar (1980), suitably modified so as to incorporate the boundary conditions (Equations 2 to 5), using a Cartesian grid composed of coordinate lines *x*, *y* at each time-step (Figure 2), with the pipe periphery approximated by straight lines.

Indicative values for the time step, Δt, which ensure stability of the explicit formulation with space step $\Delta x = \Delta y = 0.01$ m, are as follows:

For insulation:

$$\Delta t < \frac{\rho c \Delta x^2}{2k} = \frac{(110\ \mathrm{kg/m^3})(840\ \mathrm{J/kg\ °C})(0.01^2\ \mathrm{m^2})}{2 \times 0.0407\ \mathrm{W/m\ °C}} = 114\ \mathrm{s} \quad (6a)$$

For concrete:

$$\Delta t < \frac{\rho c \Delta x^2}{2k} = \frac{(2400)(880)(0.01)^2}{(2)(2.03)} = 52\ \mathrm{s} \quad (6b)$$

Although the explicit formulation could well be used, a fully implicit formulation has been employed. In most cases, space and time steps $\Delta x = \Delta y = 0.02$ m and $\Delta t = 60$ s have been selected after making grid dependence tests. The calculations were performed on a XT personal computer with 640 KB RAM. It was assumed that boundary condition 3 is periodic, i.e., the daily cycle of the ambient temperature, $T_{amb}(t)$, and the incident solar radiation, $I(t)$, are repeated on consecutive days. Therefore, the solution converged to a periodic steady-state condition, i.e., it was repeated on consecutive days.

Periodic steady-state solution in the Athens area

The periodic steady-state solution of Equation 1 with boundary conditions 2–5 has been obtained for a wide range of parameters. As an example, the solution will be given for a typical roof slab composed of three layers with thicknesses and corresponding properties given in Table 1. The values of the remaining parameters are given below.

With reference to Figure 1, the geometrical characteristics of the panel system are fixed to $S = 0.30$ m, $D = 0.02$ m, $d = 0.03$ m, and $L = 0.34$ m.

Although the ambient temperature variations and irradiation through windows may produce a marked fluctuation of interior temperature $T_a(t)$, in the examples presented in Figures 3 to 8, it has been assumed that $T_a = 24$°C and $T_w = T_a + 2 = 26$°C, for simplicity only. The calculation method does not require the assumption of a constant internal temperature, which may be any function of time.

It is not possible to define values of the heat transfer coefficients suitable for all cases, since they depend on temperature, velocity, and characteristics of the surfaces. In the examples of Figures 3 to 6, it has been assumed that $h_l = h_{r,l} = 5$ W/m^2 °C, $h_u = 16.9$ W/m^2 °C, $h_p = 3100$ W/m^2 °C, $a = 0.44$, and $\varepsilon \Delta R = 63$ W/m^2 (see ASHRAE Fundamentals 1985; ASHRAE Guide and Data Book 1962; Recknagel-Sprenger 1960; Shoemaker 1954). These values are not "suggested values," but simply values used in the example calculation presented here. For variations of the heat transfer coefficients up to ± 50 percent about the above values, the variations in the results are up to ± 20 percent.

The total solar radiation, $I(t)$, incident on the upper surface

Table 1 Depths L_i and corresponding properties ρ_i, c_i, k_i for the roof slab of Figures 3–6 (lower layer, $i = 3$)

i	Material	L_i (m)	ρ_i (kg/m^3)	c_i J/kg °C	k W/m °C
1	Cellular concrete	0.10	800	800	0.29
2	Pumice concrete	0.10	800	712	0.29
3	H.W. concrete	0.14	2400	880	2.03

Table 2 Constants in Equations 8–10

i	a_i	b_i	c_i
0	0.11048968×10^{1}	0.12321833×10^{0}	$8.51527187 \times 10^{-2}$
1	$0.62310300 \times 10^{-3}$	$-0.24593090 \times 10^{-3}$	$1.64532521 \times 10^{-4}$
2	$-0.21655676 \times 10^{-4}$	$0.13219840 \times 10^{-4}$	$1.30162335 \times 10^{-5}$
3	$0.10841363 \times 10^{-6}$	$-0.67643523 \times 10^{-7}$	$-7.27912620 \times 10^{-8}$
4	$-0.14720401 \times 10^{-9}$	$0.90926050 \times 10^{-10}$	$9.86283730 \times 10^{-11}$

CB is prescribed (see ASHRAE Fundamentals 1985) as

$$I(t) = \frac{A[C + \sin \beta(t)]}{\exp[B/\sin \beta(t)]} \tag{7}$$

where $\beta(t)$ is the time-dependent solar altitude and

A = apparent solar irradiation at zero air mass, W/m^2

B = atmospheric extinction coefficient, dimensionless

C = diffuse radiation factor, dimensionless

The values of the above quantities for the Athens area have been calculated by performing a statistical process of local solar radiation measurements of many years (Kouremenos *et al.* 1985), and the results have been correlated as

$$A = \sum_{i=0}^{4} a_i J^i, \quad kW/m^2 \tag{8}$$

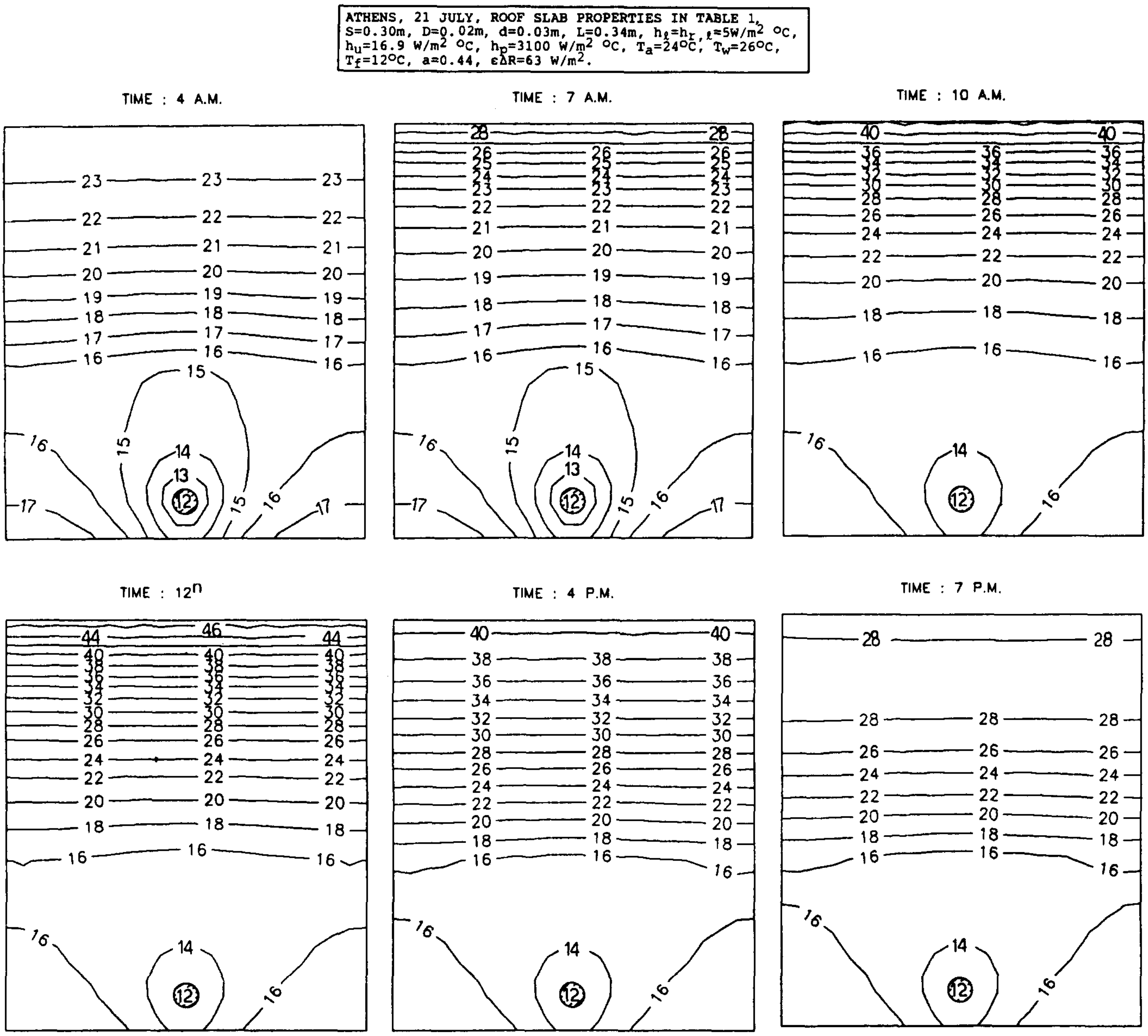

Figure 3 Temperature contours at $t = 4, 7, 10, 12, 16, 19$ hours for 21 July in Athens

Table 3 Coefficients in Equation 11

Day	M	C_1	C_2	C_3	S_1	S_2	S_3
21 Jan	9.384	−1.6730	0.7110	0.0085	−1.7230	0.5240	−0.2410
21 Feb	10.984	−2.1500	0.7860	−0.1070	−1.7500	0.5780	−0.0724
21 Mar	11.956	−1.8980	0.6510	−0.0097	−1.0840	0.2560	−0.0093
21 Apr	15.072	−2.6680	0.7270	0.2690	−1.7630	0.2430	0.2120
21 May	20.724	−2.7960	0.7590	0.0926	−1.3440	0.1190	0.3080
21 Jun	26.192	−3.3930	1.1150	0.1560	−2.1960	−0.1390	0.1570
21 Jul	28.080	−3.3590	0.8110	0.2720	−2.0770	0.1340	0.1400
21 Aug	27.880	−3.1130	0.8840	0.0670	−1.9780	0.4860	0.3610
21 Sep	23.236	−3.3100	1.0350	−0.0495	−1.6960	0.4770	0.2210
21 Oct	18.144	−2.6300	1.0850	−0.1440	−1.4270	0.6040	0.1380
21 Nov	11.968	−1.3000	0.6160	−0.2520	−0.3170	0.6290	−0.0490
21 Dec	9.264	−1.1100	0.5210	−0.1520	−0.8810	0.3330	0.0080

$$B = \sum_{i=0}^{4} b_i J^i, \quad \text{dimensionless} \tag{9}$$

$$C = \sum_{i=0}^{4} c_i J^i, \quad \text{dimensionless} \tag{10}$$

where J (= 1–365) is the day of the year and a_i, b_i, c_i are constants given in Table 2. The values of solar radiation calculated in terms of A, B, C correspond to a clear sky and are, therefore, suitable only for air-conditioning calculations, not for applications concerning collection of solar energy.

The ambient temperature, $T_{amb}(t)$, in the Athens area is prescribed as

$$T_{amb}(t) = M + \sum_{i=1}^{3} C_i \cos\left[i\frac{360}{24}(t-0.5)\right] + \sum_{i=1}^{3} S_i \sin\left[i\frac{360}{24}(t-0.5)\right] \tag{11}$$

where t (= 1–24) is the hour of the day and M, C_i, S_i are day-dependent coefficients given in Table 3 for the 21st day of each month. They have been calculated by performing a statistical process of hourly temperature measurements of many years (Kouremenos and Antonopoulos 1986).

The periodic steady-state solution obtained under the above conditions for the 21st day of July is given in Figure 3 in the form of temperature contours for the hours t = 4, 7, 10, 12, 16, 19 hours.

In order to examine the effect of the pipe spacing, S, and of the mean cooling fluid temperature, T_f, solutions have been obtained for various values of these parameters in the ranges S = 0.10 m–0.30 m and T_f = 12°C–16°C. Figure 4 shows the calculated temperature contours for 21 July at t = 12 hours, T_f = 12°C, and two values of the pipe spacing, i.e., S = 0.10 m and S = 0.20 m. Figure 5 shows the calculated contours for the same day at t = 18 hours, S = 0.20 m, and two values of the cooling fluid temperature, i.e., T_f = 12°C and T_f = 16°C.

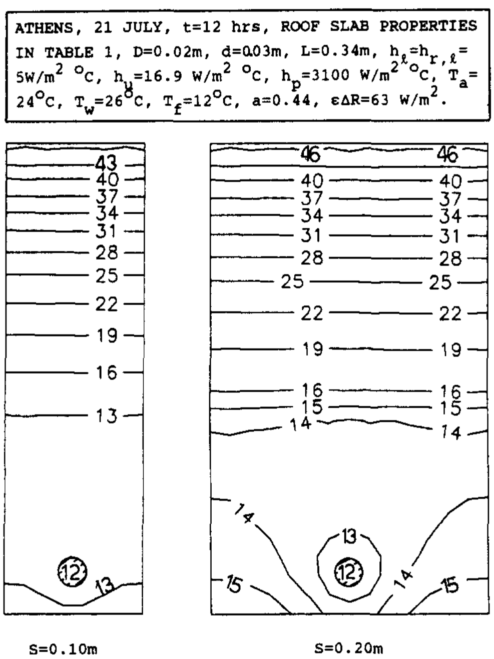

Figure 4 Temperature contours for 21 July in Athens at t = 12 hours, T_f = 12°C, and two values of the pipe spacing: S = 0.10 m and S = 0.20 m

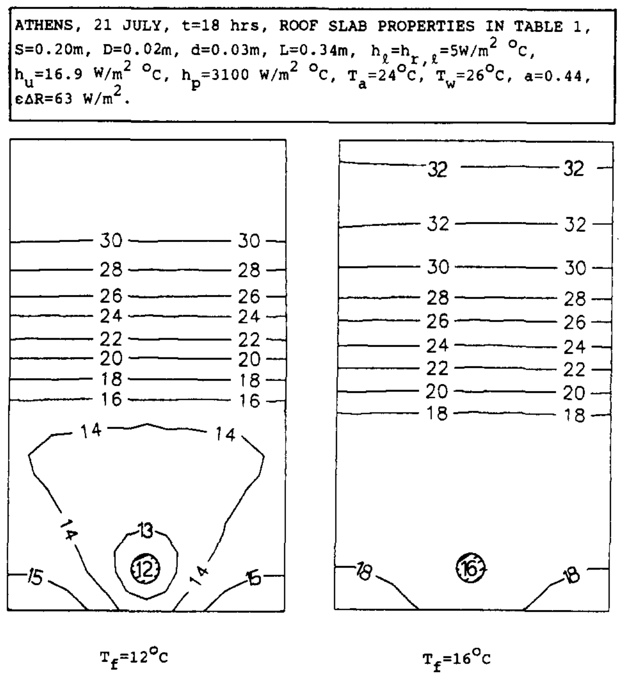

Figure 5 Temperature contours for 21 July in Athens at t = 18 hours, S = 0.20 m, and two values of the cooling fluid temperature: T_f = 12°C and T_f = 16°C

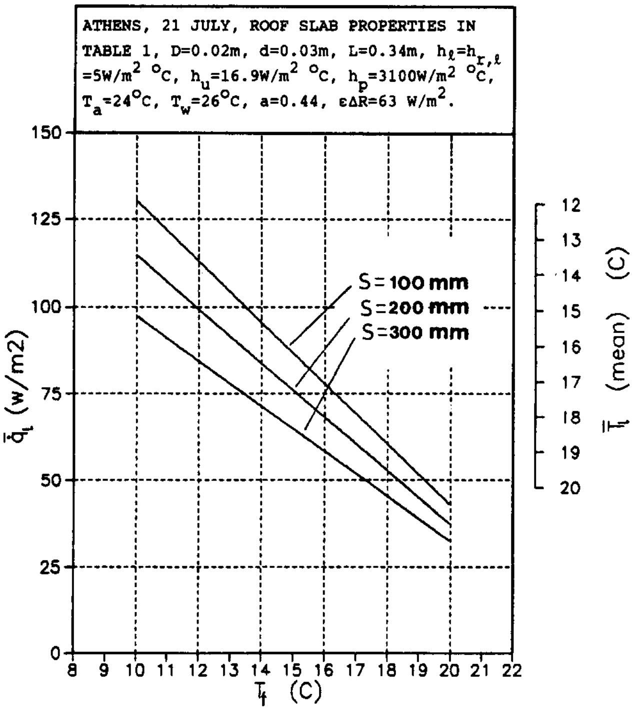

Figure 6 Periodic steady-state solution: Mean heat flow $\bar{q}_l$ absorbed by the panel from the room and mean panel temperature $\bar{T}_l$ in terms of the mean cooling fluid temperature T_f with the pipe spacing S as a parameter

From the practical point of view, it is important to know the mean (in respect to space and time) heat flow $\bar{q}_l$ absorbed by the panel from the room. This heat flow has been calculated and is given in Figure 6 in terms of T_f with S as a parameter. In the same figure, the corresponding values of the mean (in respect to space and time) panel temperature $\bar{T}_l$ are also given.

The steady-state solution

In the case of floor slabs, i.e., those separating two stories, the problem is simplified because the air temperature at the upper surface of the slab is constant and solar radiation is absent, i.e., the time-dependent terms $T_{amb}(t)$ and $I(t)$ vanish and boundary condition 3 now becomes

$$\dot{q}_u(x, L) = h_u[T_a - T_u(x, L)] + h_{r,u}[T_w - T_u(x, L)] \quad (12)$$

where

h_u = coefficient of heat transfer by convection at the upper surface of the floor slab, W/m² °C

$h_{r,u}$ = coefficient of heat transfer by radiation at the upper surface of the floor slab

T_w = mean temperature of surrounding wall surfaces

Under the above conditions, the problem becomes steady. Solutions for this case have been obtained for various values of the parameters. Examples are shown in Figures 7 and 8. The former shows temperature contours for pipe spacing $S = 0.3$ m and two values of the cooling fluid temperature, i.e., $T_f = 12°C$ and $T_f = 16°C$, while the latter shows the mean heat flow, $\bar{q}_l$, absorbed by the panel from the room and the mean panel temperature, $\bar{T}_l$, in terms of T_f with S as a parameter. These results refer to a typical panel construction with three layers,

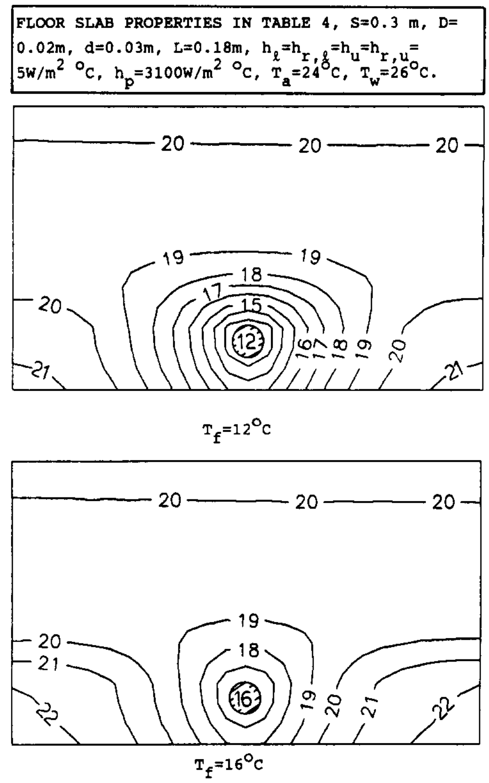

Figure 7 Steady-state solution for the typical floor slab of Table 4: Temperature contours for $S = 0.3$ m and two values of the cooling fluid temperature $T_f = 12°C$ and $T_f = 16°C$

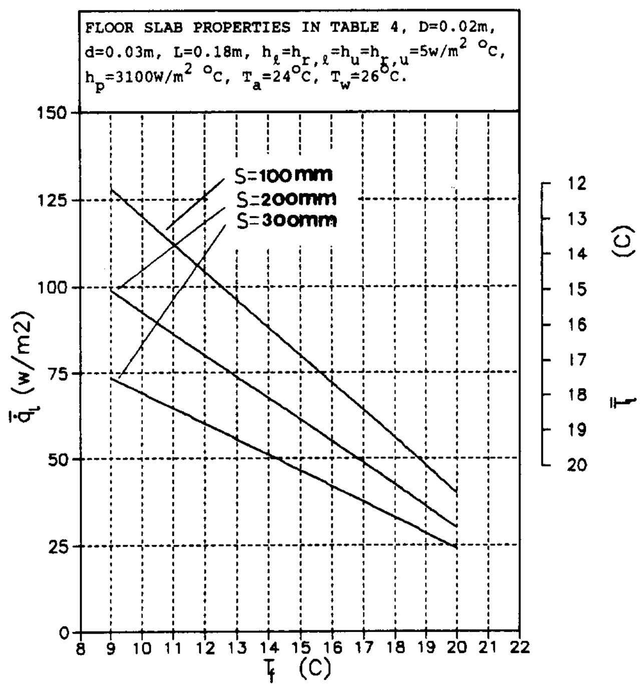

Figure 8 Steady-state solution for the typical floor slab of Table 4: Mean flow $\bar{q}_l$ absorbed by the panel from the room and mean panel temperature $\bar{T}_l$ in terms of the cooling fluid temperature T_f, with the pipe spacing S as a parameter

Table 4 Depths L_i and corresponding properties ρ_i, c_i, k_i for the floor slab of Figures 7 and 8 (lower layer $i = 3$)

i	Material	L_i (m)	ρ_i (kg/m^3)	c_i J/kg °C	k W/m °C
1	H.W. concrete	0.10	2400	880	2.03
2	Insulation	0.04	105	795	0.036
3	Concrete mixture	0.04	2300	880	1.40

the thicknesses and properties of which are given in Table 4. The heat transfer coefficients at the upper surface have been taken as $h_u = h_{r,u} = 5$ W/m^2 °C. The values of the remaining parameters are as in the examples of the previous section.

Conclusion

A method and a corresponding computer program has been developed for the calculation of panel space-cooling systems. The method is based on a finite-difference solution of the transient heat-conduction differential equation within the roof slab containing the cooling pipes, with periodic (in the general case) boundary conditions on the upper surface of the slab. The results include the temperature field within the slab, the heat fluxes on both surfaces of the slab, and also the heat absorbed by the cooling fluid for each hour of the day. Calculations are made along typical days of the year using climatological data for the Athens area.

For the typical roof-slab construction examined, the following conclusions may be drawn concerning the periodic steady-state solution.

(1) As shown in Figure 3, there is only a small change in the temperature field within the lower half of the slab, while on the upper half the temperature changes considerably, owing to the influence of the ambient temperature and the incident solar radiation. Thus, for 21 July in Athens, on the upper surface the temperature is uniform and ranges from 23°C at 4 hours to 46°C at 12 hours. For 12°C cooling-fluid temperature, on the lower surface the temperature remains unchanged with time, but it changes from 14°C at the cooling-pipe location to 17°C at the midposition between pipes.

(2) The effect of pipe spacing S on the temperature field within the roof slab is strong, as shown in Figure 4. Thus, for mean cooling fluid temperature $T_f = 12$°C, the panel surface temperature T_l remains practically uniform for pipe spacing $S = 0.10$ m, i.e., $T_l = 13$°C, while for $S = 0.20$ m it is $T_l = 14$°C–15°C. At $t = 12$ hours, the corresponding upper surface temperatures for 21 July in Athens are $T_u = 43$°C and $T_u = 46$°C, respectively.

(3) The effect of the cooling fluid temperature T_f is illustrated in Figure 5, which shows temperature contours for pipe spacing $S = 0.20$ m at $t = 18$ hours. For $T_f = 12$°C the panel temperature is $T_l = 14$°C–15°C, while for $T_f = 16$°C it is $T_l = 17$°C–18°C. The effect of T_f on the upper surface temperature is weak, i.e., a temperature difference of about 1°C is observed between the two cases.

(4) The effect of both pipe spacing, S, and mean cooling fluid temperature, T_f, on the heat absorbed by the panel from the room is shown in Figure 6 for the typical roof slab considered on 21 July in the Athens area. It is seen that the effect of S is more pronounced at the lower values of T_f. Thus, at $T_f = 12$°C, $\dot{q}_l = 85$ W/m^2–113 W/m^2 for $S = 0.30$ m–0.10 m, while for $T_f = 18$°C the corresponding values are $\dot{q}_l = 45$ W/m^2–60 W/m^2.

Similar conclusions may be drawn from Figures 7 and 8 concerning the steady-state solution for a typical floor-slab construction.

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